

Slope Stability Analysis through the Method of Total Ultimate Equilibrium

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Abstract

This paper is a review of doctoral thesis concerning some of the classical methods for the slope stability analysis and their weakness in terms of static behavior of the slope body. Newly introduced approach to the subject (the method of total ultimate equilibrium), which solves the statically indeterminate system involving all three equilibrium equations and achieves better safety factor, is explained in short, as well as other benefits comparing to the classical methods. Finally, possible directions of further method's development are indicated.

Introduction

Slope planning on open pit mines is a complex procedure that involves at least two opposite requirements:

- 1) Economic interest implies higher slope angle (to decrease the amount of waste)
- 2) On the contrary, the safety of a mine implies smaller slope angle, because the slope stability decreases with increased inclination of a slope

The stability of partial and general slopes in open pits must be guaranteed in every moment of exploitation.

Slope analysis is not limited on the problems in open pit mines. For example, these methods can also be applied on natural slopes (for example, in landslide analysis).

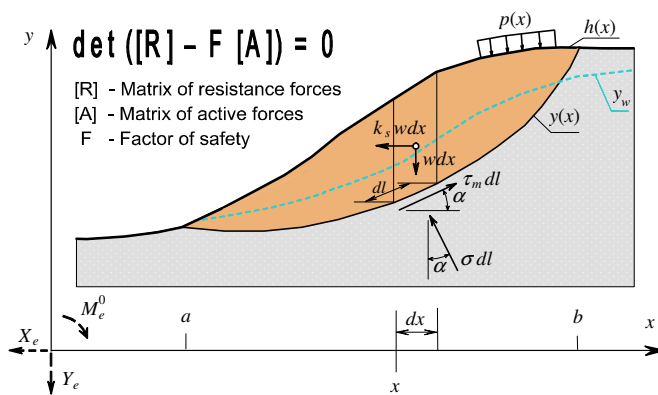


Figure 1: a slope – an illustration

Methods of Slope Stability Analysis

The methods of slope stability can be divided in two groups according to the nature of stress–strain dependency assumption:

- 1) **Rigid-plastic** (ideal plastic) material model
- 2) **All other** material model assumptions (like linear-elastic, non-linear elastic and other)

Until now, there were no mathematical models that could explain all aspects of the behavior of soil ground under the impact of pressure forces. The finite element methods are widespread in everyday engineer praxis; nevertheless, slope stability analysis methods based on the non rigid-plastic models are not utilized with same frequency as the rigid-plastic model. The reason for this situation can be easily explained: involving non-trivial material models requires complex algorithms for the slope stability analysis and must be accompanied with additional experimental tests to obtain the new material characteristics.

The presumption of the rigid-plastic material model is that the stresses in the material can increase until a limit is achieved without strains (displacements), and beyond this limit the strain will increase infinitely under constant stress conditions. If this model is accepted, mathematical modeling can be done only through the equilibrium equations and the fracture condition equation. In soil mechanics, the Mohr-Coulomb model of failure condition is widely accepted. Because the equilibrium equations are targeting the conditions where the strength of material disappears, the term limited equilibrium equation is also frequently used.

Several authors have investigated the problem of slope stability, and all of their methods are well-known and widely accepted. Relatively high number of methods in use can be explained through the statically indeterminacy. None of the classical limited equilibrium methods can satisfy all three equilibrium equations of a slope body as a whole, nor the stress results (achieved through those method's) belong to the set of statically possible solutions.

Method of Total Ultimate Equilibrium – Short Description of the Solution

The main goal of the author was to find the algorithm that produce a solution, where the state of the stress do not fall beyond the borders of the set of statically possible solutions, alongside with the satisfaction of all three equilibrium equation conditions of the slope body.

Integral-differential equations of the ultimate (limited) equilibrium contain (as unknown factors) the function of normal stress and the safety factor (one scalar value).

The unknown function of the normal stress can be represented as a Ritz sum of unknown coefficients and selected interpolation functions. Through this representation, the ultimate equilibrium equations become a homogeneous form (homogeneous considering the unknown coefficients). The safety factor can be achieved through the condition of existence a non-trivial solution.

It can be shown that the safety factor is the eigenvalue of the limited equilibrium equations in the matrix form.

The selected interpolation functions should represent the best possible approximation of the normal stress. There are no exact criterion's for this selection. Nevertheless, it was experimentally proved that the functional dependency of this selection and gained results is very weak (the influence of this selection upon the safety factor is less than one percent).

The Benefits of Total Ultimate Equilibrium Method

No other method (used to analyze the slope stability, or any other geomechanical problem) known to the author is interpreting the safety factor as the eigenvalue of a matrix equation. This approach is usual in some other disciplines, like for instance the stability of structures.

Classical methods of the slope stability analysis (like Bishop, Fellenius, Carter and Janbu) can be interpreted as special solutions of the total ultimate equilibrium method.

The safety factor (as gained through some of the classical methods) depends on the selection of the coordinate system. In the total ultimate equilibrium method, the safety factor is independent of the selected coordinate system.

All three ultimate equilibrium equations are satisfied in this new solution and equally treated in the algorithm (which does not apply to any of the classical solutions). The slope line can have any arbitrary selected shape; the conditions are that this line must be continuous, and that its first differential quotient is limited.

No extra limitation must be applied. Contrary to some of the classical methods, the quality of the result does not depend on general inclination and shape of the slip surface, nor does it depend on the slope body thickness.

Further Developing Directions of the Total Ultimate Equilibrium Method

Differential equations for any special slip surface shape can be developed from the moment ultimate equilibrium equations. In doctoral thesis, this has been done for the plain and cylindrical surface with circular generatrix. The solutions for the cylindrical surface with logarithm spiral generatrix can also be developed.

For practical reasons, only the model of Mohr-Coulomb yield conditions has been used (linear functionality). It is possible to expand the method of total ultimate equilibrium too the model with non-linear dependency of shear strength – normal stresses (Envelope of Mohr stress circles).

Most of the limited equilibrium methods is using a two dimensional (2D) model to perform the slope analysis, which has also been used in the doctoral thesis. Those classic limited equilibrium methods do not have the possibility of transforming their two dimensional approach in three dimensions (3D modeling). This kind of transformation in the total ultimate equilibrium method is

relatively straightforward: all matrices (i. e. the degree of characteristic polynomial) would increase from 3 to 6.

When comparing the results of this new method with the classical ones, it can be observed that the value of safety factor increases for the same slope models. This can be explained with the satisfaction of all three equilibrium equations (which does not apply to any of the classical solutions). The quality of gained results is at least of equal quality as it can be gained through the classical methods:

1) It can be observed, that the approach of Fellenius and Bishop is concentrated around the moment equations. For this reason, those methods will produce acceptable results under the assumption of the circular slip surface. Under those conditions, the total ultimate equilibrium method will give the results of the better quality (with remark that the new method is not limited to the circular generatrix when producing acceptable results).

2) The methods of Janbu and Carter can use an arbitrary shape of slip surface, but do not take into account the moment equation (i. e. impose the force equations as the only important equations). For this reason, they are an appropriate choice for a shallow slip surfaces with small slope angles. This special limitation also does not apply for the total ultimate equilibrium method.

Conclusion

In this paper, some of the basic characteristics of the total ultimate equilibrium method have been demonstrated, as well as the method's benefits comparing with the classical methods. The author hopes that this document have clarified some aspects of this new method and created interest to gather more information. In that case, the author is willing to provide more facts about the method, as well to discuss the possibilities of cooperation towards further theoretical development and practical applications.

References

1. Slavko M. Puljić: *Development of ultimate equilibrium method as mathematical modeling approach for slope stability analysis on open pits*, doctoral dissertation, 1992., Faculty for Mining and Geology, Belgrade, Serbia

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Appendix A

The algorithm of the total ultimate equilibrium method has been successfully implemented (on mathematical level) and used in several open pit mines in the region of the former Yugoslavia. Following graphics (and their accompanying results) has been generated semi-automatically, and are presented here as an illustration of the method.

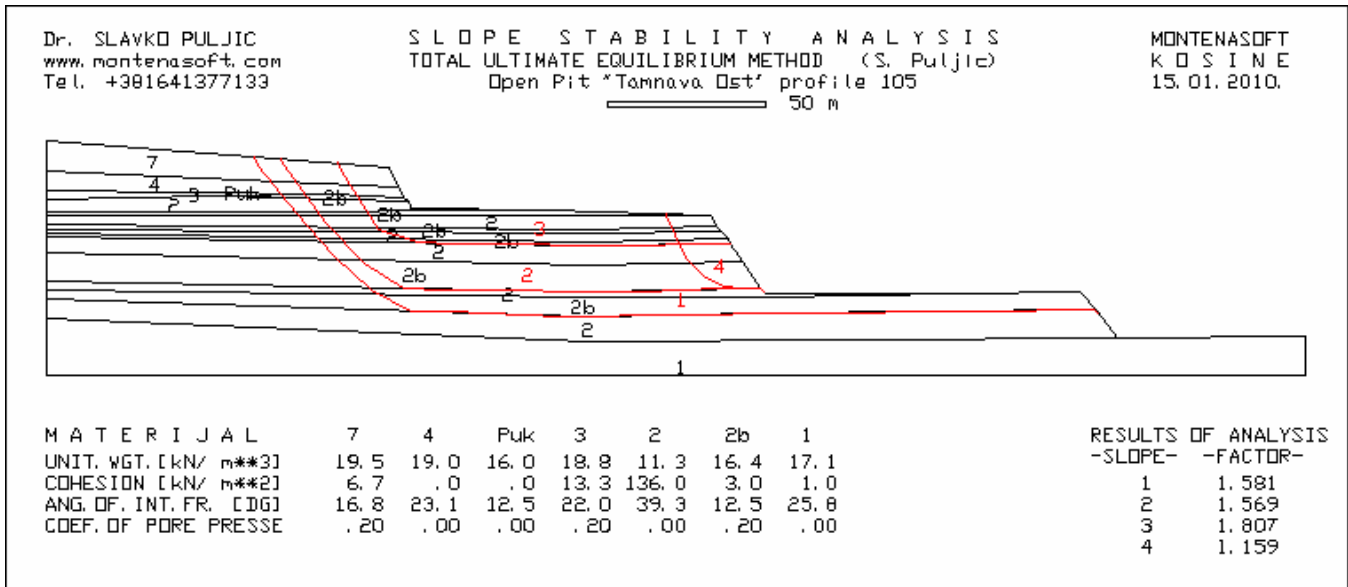


Figure A.1: Total ultimate equilibrium analysis on open pit mine “Tamnava OST”, Serbia

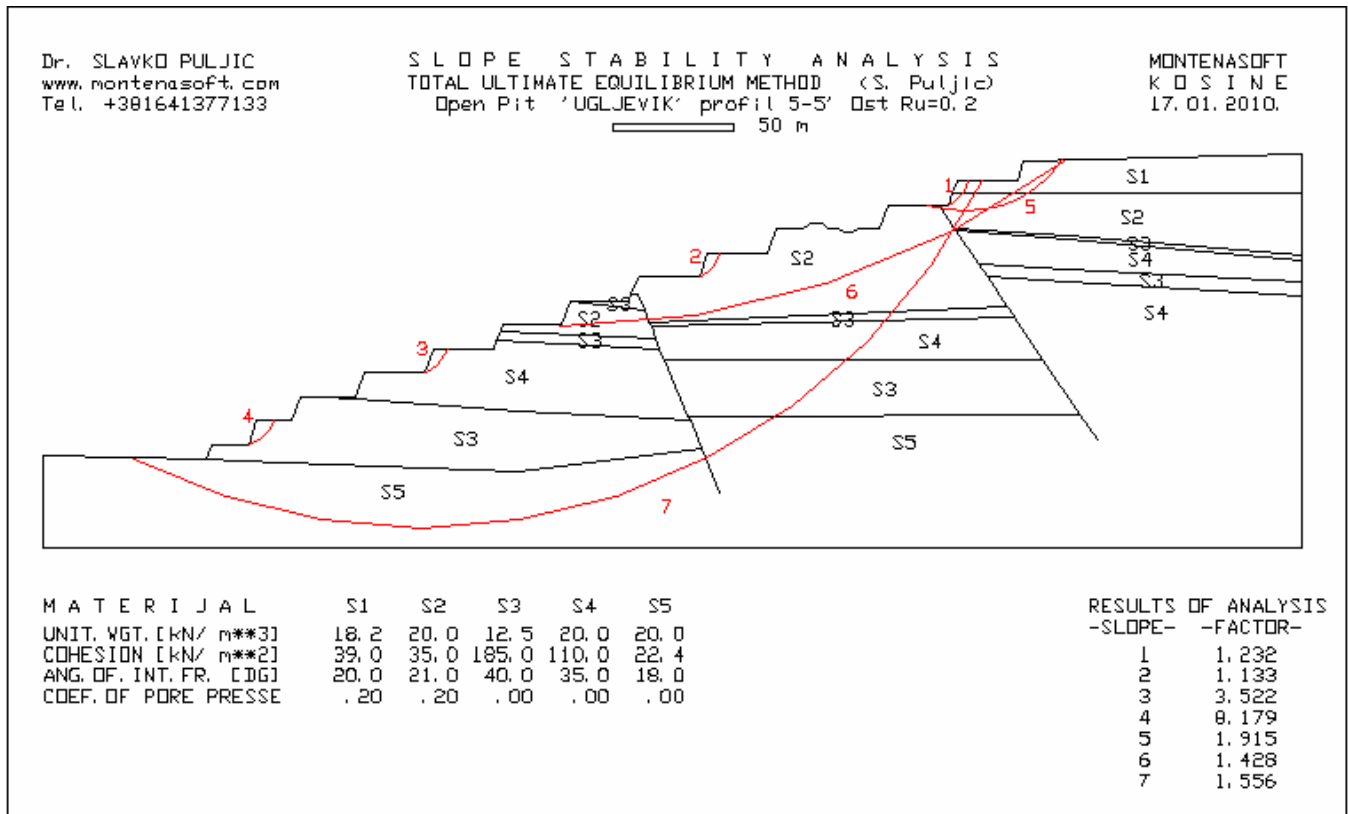


Figure A.2: Total ultimate equilibrium analysis on open pit mine “Ugljevik”, Bosnia and Herzegovina